16

DISSERTATIO ACADEMICA

THEORIAM ÆQUATIONUM FUNCTIONALIUM DUARUM VARIABILIUM EJUSQUE IN DOCTRINA SERIERUM USUM EXHIBENS;

QUAM

CONSENSU AMPLISS. FACULTATIS PHILOSOPH.

AD IMPERIALEM ACAD. ABOËNSEM,

PRÆSIDE

Mag. NATH. G. AF SCHULTÉN,

Mathematum Professore Publ. & Ord., Acad. Imperialis Scientiarum Petropolitanæ Socio Corresp.,

PRO GRADU PHILOSOPHICO

P. P.

JOHANNES FREDERICUS TICKLÉN,

Ostrobotniensis.

In Audit. Philos. die XXII Junii MDCCCXXVII. horis a. m. solitis.

P. VII.

ABOÆ, Typis FRENCKELLIANIS.

THESES.

I.

De influxu magis minusve salubri in incrementa Geometriæ Calculi analytici diversa quidem judicia ferri possunt. Nos vero laudandum illum potius quam vituperandum censemus.

II.

Ii qui contra sentiunt, afferre solent, laudatum Calculum, ob facilitatem applicationis suæ nimium sæpe in usus geometricos adhibitum, filum quasi præripuisse ipsi intuitivæ speculationi. Quod quidem neque nos infitiamur.

III.

Temerarium igitur illius usum tironibus non commendaverimus.

IV.

Ex altera autem parte negari nequit, calculum analyticum, propter directam quasi, qua incedit, viam ad resultata geometrica facile indaganda quam maxime conferre.

V.

Contendi hanc ipsam ob caussam quoque potest, territorium Geometriæ Analyseos ope valde fuisse amplificatum.

ad debitam resultatis conciliandam universalitatem ratio habenda non sit hujusmodi quantitatum k, quo fieret, uti primo res se offert intuitu, ut complicatam omnino induerent ista speciem. Ad hoc vero respondemus, in applicanda de qua agitur formula necessariam minime esse introductionem istiusmodi quantitatum k, sed sufficere omnino valori formulæ finali unicam adjecisse hujusmodi quantitatem; quod vel inde elucet, quod de determinatione jam tantum agitur formæ

$$\Sigma p_x q_x,$$

cui, secundum ea quæ allata nuperrime sunt, per simplicem tantummodo additionem quantitatis memorati generis k generalis concilianda est indoles *).

*) Quod quidem quo plenius illustretur, ponatur utique, nulla habita ad quantitates istiusmodi k ratione,

$$\Sigma q_x = r_1$$

$$\Sigma^2 q_x = r_2$$

$$\Sigma^3 q_x = r_3$$

$$\ldots\ldots;$$

unde, introducta ista quidem quantitate, habebitur

$$\Sigma q_x = r_1 + k$$

§. VII.

Theoriam æquationis simplicis 8), cui, ob insignem ejus hoc in argumento usum diutius aliquanto

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$$\Sigma^2 q_x = r_2 + \Sigma k$$

$$\Sigma^3 q_x = r_3 + \Sigma^2 k$$

$$\cdots\cdots\cdots\cdots$$

Illi quidem ipsarum

$$\Sigma q_x,\ \Sigma^2 q_x,\ \Sigma^3 q_x,\ \&c.$$

valores ad sequentem ducunt formulæ b) speciem:

$$\begin{aligned}\Sigma p_x q_x = p_x r_1 & \\ & -(p_{x+1}-p_x)(r_2+r_1) \\ & +(p_{x+2}-2p_{x+1}+p_x)(r_3+2r_2+r_1) \\ & -(p_{x+3}-3p_{x+2}+3p_{x+1}-p_x)(r_4+3r_3+3r_2+r_1) \\ & + \cdots\cdots\cdots\cdots,\end{aligned}$$

hi vero ad sequentem:

$$\begin{aligned}\Sigma p_x q_x = p_x(r_1+k) & \\ & -(p_{x+1}-p_x)(r_2+\Sigma k+r_1+k) \\ & +(p_{x+2}-2p_{x+1}+p_x)(r_3+\Sigma^2 k+2(r_2+\Sigma k)+r_1+k)\end{aligned}$$

quanto immorati sumus, jam relinquentes, æquationem ipsam generalem 7), quæ, ut eamdem repeta-

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$$-(p_{x+3}-3p_{x+2}+3p_{x+1}-p_x)(r_4+\Sigma^3 k+3(r_3+\Sigma^2 k)$$
$$+3(r_2+\Sigma k)+r_1+k)$$
$$+\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.$$

$$=p_x r_1$$
$$-(p_{x+1}-p_x)(r_2+r_1)$$
$$+(p_{x+2}-2p_{x+1}+p_x)(r_3+2r_2+r_1)$$
$$-(p_{x+3}-3p_{x+2}+3p_{x+1}-p_x)(r_4+3r_3+3r_2+r_1)$$
$$+\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.$$
$$+p_x k$$
$$-(p_{x+1}-p_x)(\Sigma k+k)$$
$$+(p_{x+2}-2p_{x+1}+p_x)(\Sigma^2 k+2\Sigma k+k)$$
$$-(p_{x+3}-3p_{x+2}+3p_{x+1}-p_x)(\Sigma^3 k+3\Sigma^2 k+3\Sigma k+k)$$
$$+\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.$$

Differentia harum expressionum, quæ est

$$p_x k$$
$$-(p_{x+1}-p_x)(\Sigma k+k)$$

petamus, sequens habetur

$$y_{x+n}+py_{x+n-1}+qy_{x+n-2}+\dots ty_{x+1}+u=o \dots 7),$$

cujusque ipsa 8) casum tantum constituit valde parti-

$$+(p_{x+2}-2p_{x+1}+p_x)(\Sigma^2 k+2\Sigma k+k)$$

$$-(p_{x+3}-3p_{x+2}+3p_{x+1}-p_x)(\Sigma^3 k+3\Sigma^2 k+3\Sigma k+k)$$

$$+\dots\dots\dots\dots,$$

primo quidem intuitu, præter quantitates ejusdem generis ac k, ipsam quoque complecti variabilem x videretur: apparenter vero tantum id fieri, sive sponte in ea istam evanescere variabilem, ex allata jam supra ratione concludendum est. Quod quidem in casu quolibet particulari directe quoque confirmare facile est. Habentur scilicet in genere

$$\Sigma k = kx + k',$$

$$\Sigma^2 k = k\Sigma x+\Sigma k' = k(\tfrac{1}{2}x^2-\tfrac{1}{2}x)+k'x-k''$$

$$=\tfrac{1}{2}kx^2+(k'-\tfrac{1}{2}k)x+k'',$$

$$\Sigma^3 k=\tfrac{1}{2}k\Sigma x^2+(k'-\tfrac{1}{2}k)\Sigma x+\Sigma k''$$

$$=\tfrac{1}{2}k(\tfrac{1}{3}x^3-\tfrac{1}{2}x^2+\tfrac{1}{6}x)+(k'-\tfrac{1}{2}k)(\tfrac{1}{2}x^2-\tfrac{1}{2}x)+k''x+k'''$$

$$=\tfrac{1}{6}kx^3+\tfrac{1}{2}(k'-k)x^2+(k''-\tfrac{1}{2}k'+\tfrac{1}{3}k)x+k''',$$

$$\Sigma^4 k=\tfrac{1}{6}k\Sigma x^3+\tfrac{1}{2}(k'-k)\Sigma x^2+(k''-\tfrac{1}{2}k'+\tfrac{1}{3}k)\Sigma x+\Sigma k'''$$

particularem, immediate considerandam nobis sumamus. Cum tantis vero, ubi vidimus, prematur diffi-

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$$=\frac{1}{6}k\left(\frac{1}{4}x^4-\frac{1}{2}x^3+\frac{1}{4}x^2\right)+\frac{1}{2}(k'-k)\left(\frac{1}{3}x^3-\frac{1}{2}x^2+\frac{1}{6}x\right)+\left(k''-\frac{1}{2}k'\right.$$
$$\left.+\frac{1}{3}k\right)\left(\frac{1}{2}x^2-\frac{1}{2}x\right)+k'''x+k''''$$
$$=\frac{1}{24}kx^4+\left(\frac{1}{6}k'-\frac{1}{4}k\right)x^3+\left(\frac{1}{2}k''-\frac{1}{2}k'+\frac{11}{24}k\right)x^2+\left(k'''-\frac{1}{2}k''\right.$$
$$\left.+\frac{1}{3}k'-\frac{1}{4}k\right)x+k'''',$$

&c. &c.

quibus quidem valoribus in allata nuper differentia substitutis, res de qua quæstio est immediate comprobabitur. Sic posito v. gr.

$$p_x=x^3,$$

abibit memorata sæpe expressio in

$$x^3 k$$
$$-(3x^2+3x+1)(kx+k'+k)$$
$$+(6x+6)\left(\frac{1}{2}kx^2+\left(k'+\frac{3}{2}k\right)x+k''+2k'+k\right)$$
$$-6\left(\frac{1}{6}kx^3+\left(\frac{1}{2}k'+k\right)x^2+\left(k''+\frac{5}{2}k'+\frac{11}{6}k\right)x+k'''+3k''\right.$$
$$\left.+3k'+k\right),$$

quæ, evoluta, fit

$$-6k'''-12k''-7k'-k,$$

ubi x non amplius occurrere videmus.

difficultatibus resolutio æquationis simplicissimæ 8), eo minus quidem universaliora quædam ipsam 7) resolvendi præcepta exspectanda sunt. Generalissima, quam hoc respectu adferendam habemus, regula, sequenti continetur calculi præscripto, quo partim ad aliam paullo simpliciorem, partim ad tractatam jam ipsam 8), resolutio hujusce æquationis revocari potest, cujusque applicatione omnia fere nituntur, quæ æquationem de qua agitur 7) tractandi nobis constant præcepta.

Posita scilicet æquatione

$$z_{x+n} + pz_{x+n-1} + qz_{x+n-2} + \ldots tz_x = o \ldots 9);$$

quæ ab ipsa 7) eo tantum differt, quod terminum hujus ultimum u non comprehendat, designantibusque

$${}^1z,\ {}^2z,\ {}^3z \ldots {}^nz$$

functiones quascumque particulares ipsius x æquationi 9) satisfacientes (i. e. substitutione earum hac in æquatione pro ipsa z_x identicam eamdem in genere efficientes), assumtisque demum

$$w_1,\ w_2,\ w_3, \ldots w_n$$

functionibus ipsius x numero n per æquationes totidem sequentes determinandis

${}^1z_{x+1}$

$$\left.\begin{array}{l}
{}^{1}z_{x+1}\,w_1 + {}^{2}z_{x+1}\,w_2 + {}^{3}z_{x+1}\,w_3 + \ldots {}^{n}z_{x+1}\,w_n = o\\
{}^{1}z_{x+2}\,w_1 + {}^{2}z_{x+2}\,w_2 + {}^{3}z_{x+2}\,w_3 + \ldots {}^{n}z_{x+2}\,w_n = o\\
{}^{1}z_{x+3}\,w_1 + {}^{2}z_{x+3}\,w_2 + {}^{3}z_{x+3}\,w_3 + \ldots {}^{n}z_{x+3}\,w_n = o\\
\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\\
{}^{1}z_{x+n-1}\,w_1 + {}^{2}z_{x+n-1}\,w_2 + {}^{3}z_{x+n-1}\,w_3 + \ldots {}^{n}z_{x+n-1}\,w_n\ o\\
{}^{1}z_{x+n}\,w_1 + {}^{2}z_{x+n}\,w_2 + {}^{3}z_{x+n}\,w_3 + \ldots\ z_{x+n}\,w_n + u = o
\end{array}\right\}\ldots f),$$

habebitur in genere:

$$y_x = {}^{1}z_x\,\Sigma w_1 + {}^{2}z_x\,\Sigma w_2 + {}^{3}z_x\,\Sigma w_3 + \ldots {}^{n}z_x\,\Sigma w_n \ldots g).$$ *)

Posito

*) Abeunte, per valores particulares

$$n = 1,\ p = o,\ q = o,\ \ldots t = -1,\ u = -v,$$

æquatione generali 7) in simplicem

$$y_{x+1} - y_x - v = o\ \ldots 8),$$

mutabitur ipsa 9) in

$$z_{x+1} - z_x = o,$$

in qua cum obtineat v. gr.

$${}^{1}z_x = 1,$$

Posito quidem

$$u = o,$$

i. e. abeunte æquatione 7) in

$$y_{x+n} + py_{x+n-1} + qy_{x+n-2} + \ldots ty_x = o \ldots 10),$$

perspicuum per theoriam eliminationis est, haberi

$$w_1 = o, \; w_2 = o, \; w_3 = o, \; \ldots \; w_n = o,$$

hincque fieri

$$\Sigma w_1 = k_1, \; \Sigma w_2 = k_2, \; \Sigma w_3 = k_3, \; \ldots \; \Sigma w_n = k_n,$$

ubi

æquationes ipsæ f) hoc in casu in

$$^1z_{x+1} \cdot w_1 - v = o,$$

i. e.

$$w_1 - v = o,$$

abibunt, unde fiet

$$w_1 = v,$$

hincque

$$y_x = \Sigma v,$$

unde perspicitur, allatam nuperrime regulam generalem ipsam quoque, quæ tractata hactenus est, æquationem 8), particularis instar casus, comprehendere.